

# Using high-spin data to constrain spin-orbit term and spin-fields of Skyrme forces

## The need to unify the time-odd part of the local energy density functional

W. Satuła<sup>1,2,a</sup>, R. Wyss<sup>2</sup>, and H. Zduńczuk<sup>1</sup>

<sup>1</sup> Institute of Theoretical Physics, University of Warsaw, ul. Hoża 69, PL-00 681 Warsaw, Poland

<sup>2</sup> KTH (Royal Institute of Technology), AlbaNova University Centre, S-106 91 Stockholm, Sweden

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**Abstract.** A method to study spin-fields and the spin-orbit potential within the local energy density approach is presented. The concept utilizes the intrinsic simplicity of terminating states in order to constrain certain parameters of the local nuclear energy functional. In particular, constraints on the isoscalar Landau parameter  $g_0$  and the strength of the spin-orbit potential are thoroughly discussed.

**PACS.** 21.60.Jz Hartree-Fock and random-phase approximations

## 1 Introduction

The terminating states are one of *the purest* examples of unperturbed single-particle (sp) motion. Hence, they are perfectly suited for unpaired Hartree-Fock (HF) calculations and, in turn, offer an excellent playground for testing and constraining various aspects of the effective NN interaction or local energy density functional (LEDF).

In this contribution, see refs. [1,2] for details, we present calculations of the energy differences,  $\Delta E$ , between terminating states within  $d_{3/2}^{-1} f_{7/2}^{n+1}$  and  $f_{7/2}^n$  configurations in  $20 \leq Z < N \leq 24$  nuclei, where  $n$  denotes the number of valence particles outside the  $^{40}\text{Ca}$  core. The value of  $\Delta E$  is dominated by the size of the magic gap 20,  $\Delta e_{20}$ . One can establish a hierarchy of the different contributions to  $\Delta e_{20}$  guided by the Nilsson model expression  $\Delta e_{20} = \hbar\omega_0(1 - 6\kappa - 2\kappa\mu)$ . Indeed, it shows that: i) flat-bottom and surface effects,  $\mu \sim 0$ , are marginal in light nuclei ii) even small changes to low-energy nuclear physics energy scale,  $\hbar\omega_0$ , which is well established, is rather unlikely since it will impair the in general good agreement between theory and experiment, in particular, in heavy nuclei. Hence, the uncertainties in  $\Delta e_{20}$  are predominantly related to the uncertainties in the  $ls$ -term enabling, in turn, a fine-tuning of its strength. This argumentation is general and pertains also to self-consistent approaches including, in particular, the Skyrme-HF (SHF) method used in this work.

We will also demonstrate that thanks to the intrinsic simplicity of the terminating states one can reduce the

arbitrariness of the time-odd (TO) channel in the Skyrme-force (SF) induced LEDF (S-LEDF) by establishing a firm constraint on the isoscalar Landau parameter  $g_0$ .

## 2 The spin-fields

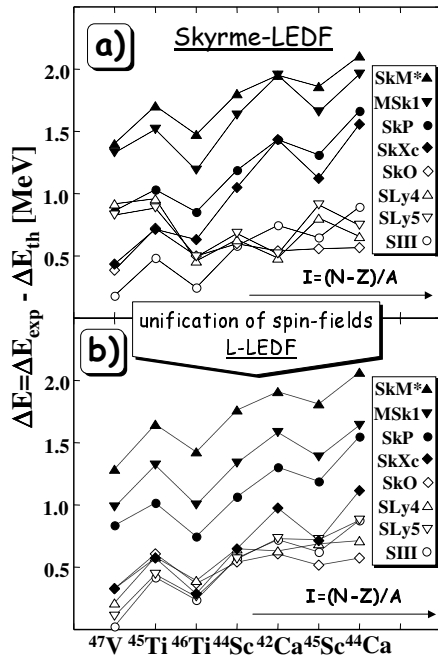
Since the SF is fitted ultimately to the time-even (TE) channel, the TO components of the S-LEDF which are not related to the TE channel through the local gauge invariance appear to be completely accidental, see refs. [1,3]. This pertains to the spin-field coupling constants  $C_{\mathbf{t}}^s$  and  $C_{\mathbf{t}}^{\Delta s}$  in the TO part of the S-LEDF:

$$\mathcal{H}_{\mathbf{t}}^{(\text{TO})}(\mathbf{r}) = C_{\mathbf{t}}^s \mathbf{s}_{\mathbf{t}}^2 + C_{\mathbf{t}}^{\Delta s} \mathbf{s}_{\mathbf{t}} \Delta \mathbf{s}_{\mathbf{t}} + C_{\mathbf{t}}^T \mathbf{s}_{\mathbf{t}} \cdot \mathbf{T}_{\mathbf{t}} + C_{\mathbf{t}}^{j:2} \mathbf{j}_{\mathbf{t}}^2 + C_{\mathbf{t}}^{\nabla j} \mathbf{s}_{\mathbf{t}} \cdot (\nabla \times \mathbf{j}_{\mathbf{t}}), \quad (1)$$

where  $\mathbf{t}$  denotes isospin. The definition of the local densities and relations between coupling constants  $C$  and the auxiliary SF parameters can be found, for example, in [4].

The uncertainty in the spin-fields is reflected in fig. 1a, showing the calculated energy differences  $\Delta E_{\text{th}}$  relative to the experimental data  $\Delta E_{\text{exp}}$ . Indeed, even a small generalization of the S-LEDF by replacing the SF-induced coupling constants  $C_{\mathbf{t}}^s$  with the following set of the Landau parameters (L-LEDF) recommended in ref. [3]:  $g_0 = 0.4$ ,  $g'_0 = 1.2$  and  $g_1 = -0.19$ ,  $g'_1 = 0.62$ , and by setting  $C_{\mathbf{t}}^{\Delta s} \equiv 0$  provides a very consistent description of the experimental data by most of the tested parameterizations, as shown in fig. 1b. Most of the parameterizations deviate from the data by  $\sim 10\%$  ( $\overline{\Delta E} \approx 500$  keV). Only for SkP, MSk1, and SkM\* it is unacceptably large,  $\sim 20\text{--}30\%$ . The

<sup>a</sup> Conference presenter; e-mail: [satula@fuw.edu.pl](mailto:satula@fuw.edu.pl)



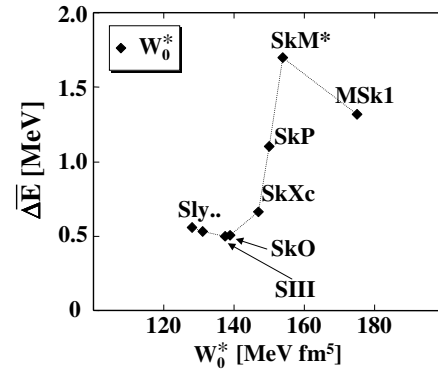
**Fig. 1.** Calculated energy differences for terminating states  $\Delta E_{\text{th}} = E[d_{3/2}^{-1}f_{7/2}^{n+1}] - E[f_{7/2}^n]$  relative to the experimental data  $\Delta E \equiv \Delta E_{\text{exp}} - \Delta E_{\text{th}}$ . Upper part shows SHF calculations for various parameterizations while lower part illustrates calculations using a unified description of the spin-fields.

detailed analysis shows that the value of  $\overline{\Delta E}$  cannot be reduced by further tuning of  $g_0$ , and that the *optimal* value of  $g_0$  deduced from our calculations is only slightly larger than the value recommended in the literature [5].

### 3 The spin-orbit term

The average discrepancy  $\overline{\Delta E}$  does not correlate directly with the bare isoscalar strength  $W_0$  of the  $\ell s$ -term. However, the value of  $\overline{\Delta E}$  correlates nicely with the isoscalar-effective-mass scaled isoscalar strength of the spin-orbit term  $W_0^* \equiv \frac{m^*}{m} W_0$ , see fig. 2, which takes into account non-local effects. Indeed, the Skyrme forces having  $W_0^* \approx 135 \pm 10 \text{ MeV fm}^5$  give similar level of agreement between theory and the data of the order of 10%. It can be shown that, by reducing the strength of the  $\ell s$ -term by 5%, the deviation from the data can be lowered to below 5%. Let us also observe that the SF giving large disagreement to the data have  $W_0^* > 150 \text{ MeV fm}^5$ . These SF cannot be *corrected* just by fine tuning of  $W_0$ .

Further detailed study shows that our calculations give rather clear preference for the non-standard parameterizations of the  $\ell s$ -term with a strong isovector dependence characterized by the ratio of the isovector to the isoscalar coupling constants  $W_1/W_0 \sim -1$ . Indeed, such forces tend to reduce the slope of  $\Delta E$  versus the reduced isospin  $I = (N - Z)/A$  which is clearly seen in fig. 1. More detailed discussion can be found in refs. [1, 2]. Let us point out that the ratio  $W_1/W_0 \sim -1$  is inspired by the rela-



**Fig. 2.** The values of  $\Delta E$  averaged over all nuclei considered in fig. 1,  $\overline{\Delta E}$ , as a function of the isoscalar strength  $W_0^*$  for different parameterizations of the SF.

tivistic models and seems to be more consistent with the data particularly in neutron rich nuclei [6, 7].

### 4 Summary

The objective of this analysis is to demonstrate that the terminating states, due to their intrinsic simplicity, offer a unique and so far unexplored opportunity to study different aspects of the effective NN interaction or nuclear local energy density functional within the self-consistent approaches.

First of all our work demonstrates that the terminating states offer a unique playground for studying the TO components of the LEDF, allowing to set a firm constraint on the isoscalar Landau parameter  $g_0$ .

Furthermore, it is shown that the  $\sim 10\%$  disagreement between theory and the data correlates nicely with the isoscalar-effective-mass scaled isoscalar spin-orbit strength. A 5% reduction of  $W_0$  appears to reduce the average discrepancy,  $\overline{\Delta E}$ , well below  $\sim 5\%$ . Our calculations give also certain preference for non-standard parameterizations of the spin-orbit term having  $W_1/W_0 \sim -1$ .

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